USE OF THE SINGLE CORE MAGNETIC
AMPLIFIER IN A MAGNETIC TIMER CIRCUIT

ROY B. WEAVER

USE OF THE SINGLE CORE MAGNETIC AMPLIFIER IN A MAGNETIC TIMER CIRCUIT

by

Roy Babbitt Weaver

Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
Monterey, California

1955

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

ENGINEERING ELECTRONICS

from the

United States Naval Postgraduate School

PREFACE

Use of the single core magnetic amplifier in a magnetic timer led to an exploration of the theoretical and experimental characteristics of this circuit. Results show not only that magnetic amplifiers can be used to control relay circuits, but also point to further extension of their field of application.

The author desires to express his appreciation for the assistance and advice given by Mr. Kenneth Goodman and Mr. Joseph Chun of the Pacific Division of Aerovox, Inc.*, in Monrovia, California, at whose plant the experimental work undertaken in connection with this thesis was performed during early 1955.

* Formerly Acme Electronics, Inc.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS AND ABBREVIATIONS

ac	Alternating current
ave	Average
e_{ac}	Instantaneous value of ac voltage applied to gate circuit
e'_{ac}	Instantaneous value of ac voltage applied to the control circuit
E_{ac}	Peak value of ac voltage applied to the gate circuit
E'_{ac}	Peak value of ac voltage applied to the control circuit
E_o	Peak value of output voltage (across the load in the gate circuit)
e_s	Instantaneous value of the signal voltage
E_s	Peak value of the signal voltage
e_I	Instantaneous voltage across the gate winding of the M/A
e_{II}	Instantaneous voltage across the control winding of the M/A
f	Frequency of applied ac voltage (400 cycles per second)
G	Gain of M/A stage
H_c	Coercive force per inch
i_L	Instantaneous load current
I_L	Peak load current
K	Peak voltage per turn
L	Mean length of flux path
M/A	Magnetic Amplifier
mmf	Magnetomotive force
ms	Millisecond

n	Number of cycles after application of signal voltage
N	Turns ratio of M/A
N_c	Number of turns in control winding
N_g	Number of turns in gate winding
R_{ct}	Total control circuit resistance
R_L	Load resistance
R_R	Load circuit rectifier resistance
t	time
t_n, t_1, etc.	time of firing during the nth, 1st, etc. half cycle
V	Volts
Z_x	Coupling impedance of ring counter circuit
	Flux change during a half cycle of supply voltage
ϕ	Flux in the M/A core
ϕ_s	Saturation or maximum flux
ϕ_r	Remanent or residual flux
θ_n	Firing angle during the nth half cycle

CHAPTER I

INTRODUCTION TO MAGNETIC AMPLIFIERS

1. Early history.

Although the magnetic amplifier, for which we will use the symbol M/A in this paper, has only recently come into real prominence in electronic equipment design, it is far from a new device. Perhaps the earliest article in the literature [11] on the saturable reactor, which is the basic element of all M/A was published by the well known English scientist, Lord Rayleigh, [17] in 1887. The first use of the term M/A to describe a device which amplified a signal by magnetic means was by E. F. W. Alexanderson [1] in 1916.

The first American patent in the saturable reactor field [11] was issued to C. F. Burgess and B. Frankenfield in 1903. However, the first patent for a saturable device whose inventor claimed "gain" properties, thereby making it a real amplifier, was that issued to the above mentioned E. F. W. Alexanderson on 28 November, 1916. Interest in the M/A as evidenced both by numbers of patent applications filed, and by articles in the literature, remained lukewarm until the late 1930's. It had just begun to intensify when the Second World War forced the channeling of available research facilities into the development of immediately producible war equipment, which in this country was almost entirely based on vacuum tube circuits.

2. Recent advances.

After the war research and development in the field of M/A devices was greatly accelerated by at least three factors. First, we had a graphic example of their potentialities when we found out the uses to which M/A had been put by the Germans, notably in their ship-board fire control systems such as that of the cruiser, Prinz Eugen [3]. Not only had they been used successfully, but records showed that they had required no servicing over a 10 year period.

Second, by the end of World War II electronic equipment had come to be so essential in both military and civilian life, and had been adapted to the performance of so many important functions, that dependability was paramount. At the same time, the increasing circuit complexity and the large numbers of components, especially vacuum tubes, which had made possible the performance of these many important functions also made the attainment of the requisite reliability very difficult. The electron tube, even when operated well within its ratings, has a considerably higher failure rate than most other circuit components, and when many tubes are required in an equipment the chances of failure are increased not only by inexorable mathematics, but also by the increased difficulty of maintaining normal temperatures when so much filament power is being dissipated as heat.

Third, the great advancements in materials available for cores and rectifiers made possible better performance with standard circuits and allowed the development of some new

circuits not previously practical. Consequently, the adaptability of M/A to various uses in the electronics field, as well as its already established use in the control and regulation of electrical power devices was greatly increased. Core materials have progressed far from the transformer grade steel of the early days. Grain-oriented silicon steels (3% Si, 97% Fe) under the trade names of Hypersil, Trancor XXX or Silectron made possible greatly improved gain [2]. Other metallurgical advances led to production of Mumetal (75% Ni, 2% Cr, 5% Cu, 18% Fe) and Supermalloy or 4-79 Permalloy (79% Ni, 4% Mo, 16% Fe) both possessing properties which further increased the range of usefulness of saturable reactors. Finally the development of a high saturation flux density material with narrow rectangular hysteresis loops made practical the single core M/A circuit with which this paper is concerned. This is grain oriented 50% Ni, 50% Fe usually in the form of tape wound toroidal cores and called Deltamax, Hypernik V, Orthonik, Permeron, or Orthonol depending on the manufacturers. Rectifier characteristics have also been greatly improved over the years. Selenium rectifier ratings have increased from 14 V. rms to 50 V. rms per cell [2]. Rectifiers for operation at much higher ambient temperatures (up to 75°C) with a life expectancy of 5,000 to 20,000 hours are now available, with other types suitable for temperatures up to 35°C which have a life expectancy of 20,000 to 100,000 hours. Germanium crystal diodes of point contact and junction types are also available

and play an important role in M/A circuits in which currents are not large.

3. Advantages.

These improved materials and the greater variety of circuits which they make possible have not, in general, given M/A any new advantages over other types of amplifiers. The jobs done by M/A devices, old or new, can usually be performed by vacuum tube or gas tube circuits, or both. What the advancements in the M/A art have done is to make this device satisfactorily adaptable to many more uses so that its natural advantages can be exploited. The most important of these advantages are reliability and efficiency. As noted above these qualities have become more and more essential as equipments become more complex. The reliability of M/A is high because of their physical ruggedness, and their electrical stability. Being composed of metallic cores, insulated copper wire, and rectifiers, all of which can be rigidly mounted as a unit and hermetically sealed against dust and moisture if desired, they are very resistant to shock or corrosion. Electrically the rectifier is the limiting element, but it is very reliable unless temperatures are extreme. The efficiency of the M/A as compared to the vacuum tube is greater principally because the M/A does not require a heated filament. The small amount of heat to be dissipated, in addition to enabling the M/A circuit to be easily potted or sealed and thus aid in increasing reliability as noted above, also eliminates the necessity for many of the blowers,

extra ventilation ducts, and heat resistant materials usually required in multitube equipments, and decreases the overall power supply drain.

4. Classifications.

M/A are classified [12] in two basic groups. Class A includes devices in which the ac power is applied to the core via the load or reactance winding. Class B includes devices in which the ac power is applied to the core via a separate primary winding. In other words M/A of the first class are saturable reactors controlled by signal mmf, while those of the second class are saturable transformers controlled by signal mmf. The saturable transformer class is further divided into types in which control is effected by varying the incremental permeability, and those in which control is accomplished by changing the coupling between the primary and secondary windings, usually by varying the reluctance of a magnetic path shunting the transformer by linking the secondary and primary windings.

The single core M/A is an example of the Class A group, although its two windings give it the appearance of a transformer when seen in schematic form. This circuit was developed by R. A. Ramey, then of the Naval Research Laboratory, and its description was first published in 1951 [15]. This paper will describe the operation of the Ramey M/A and give specific design data and experimental results for an application of the circuit in a magnetic delay line or timer, producing voltage

or current pulses which actuate relays in a precisely timed sequence.

CHAPTER II

THEORY OF THE SINGLE CORE M/A

1. Distinguishing features.

According to its inventor, the single core M/A is a development resulting from consideration of the M/A as a voltage sensitive device rather than a current sensitive device. Though this has not been the usual approach, at least in the past, it seems a logical one. Looking at a typical flux density versus mmf curve (hysteresis loop) such as that shown in Figure 1 it is evident that the flux density is not uniquely determined by the mmf, which is a function of the current, except during periods when the core is saturated. During the very important part of the operating cycle in which the core is not saturated, the flux density may have any value between the points at which the loop intersects the vertical line representing the mmf (or current) at the moment. For instance, at an mmf equal to "a" on Figure 1 the flux density may vary from "a" to "a " according to the history of the voltage applied to the core. Even at 0 mmf (0 current) the flux density may have any value from that of residual induction or flux, Br, in one direction to the same value in the opposite direction. The factor which actually determines the flux is the time integral of the voltage (volt-seconds) which has been applied to the core since it was last saturated. The control voltage is the only truly independent variable.

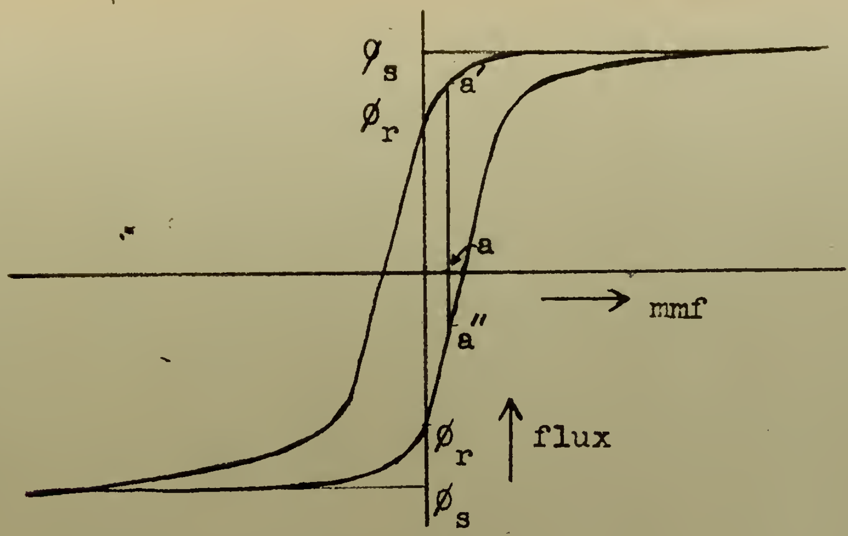

Figure 1 Typical Magnetization Curve

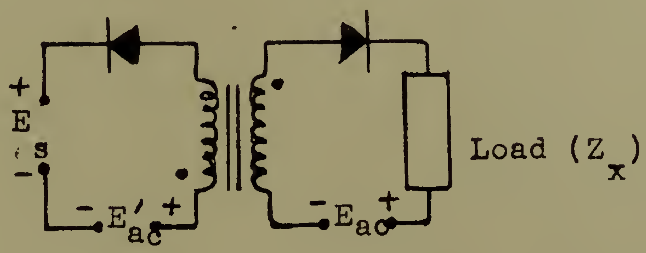

Figure 2(a) Basic Single Core M/A Circuit

(Two Winding)

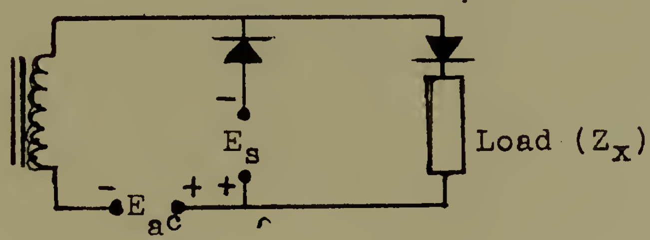

Figure 2(b) Basic Single Core M/A Circuit

(One Winding)

The circuit Mr. Ramey developed as a result of this approach is shown in Figure 2. There are two versions, the double winding type of Figure 2 (a) and a less versatile variation, the single winding type shown in Figure 2 (b). Though operation is basically the same, the double winding type will be considered henceforth. It consists of a magnetic core coupling two circuits via two windings. One is the load or gate circuit, consisting of a source of alternating voltage E_{ac}, a rectifier, and a load in series with one winding. The other is the control circuit, consisting of a source of alternating voltage E'_{ac}, a rectifier, and a control voltage in series with the second winding of the core. E_{ac} and E'_{ac} have the same frequency and phase. Their magnitudes have a ratio equal to the turns ratio of the saturable reactor windings, and their polarities are as noted on the figure. The dots refer to winding polarity.

Although two of these M/A may be connected in push-pull as shown in Figure 3 to give full wave output, it is perfectly feasible to use the one core and two windings of Figure 2 (a). The output will be half wave rectified ac whose average value is almost exactly proportional to the control or signal voltage. The usual M/A circuit, one example of which is the series type shown in Figure 4, required two cores so that voltages which are induced in the control windings through transformer action by the gate windings while the core is unsaturated may be balanced out. It is from this distinguishing feature that the

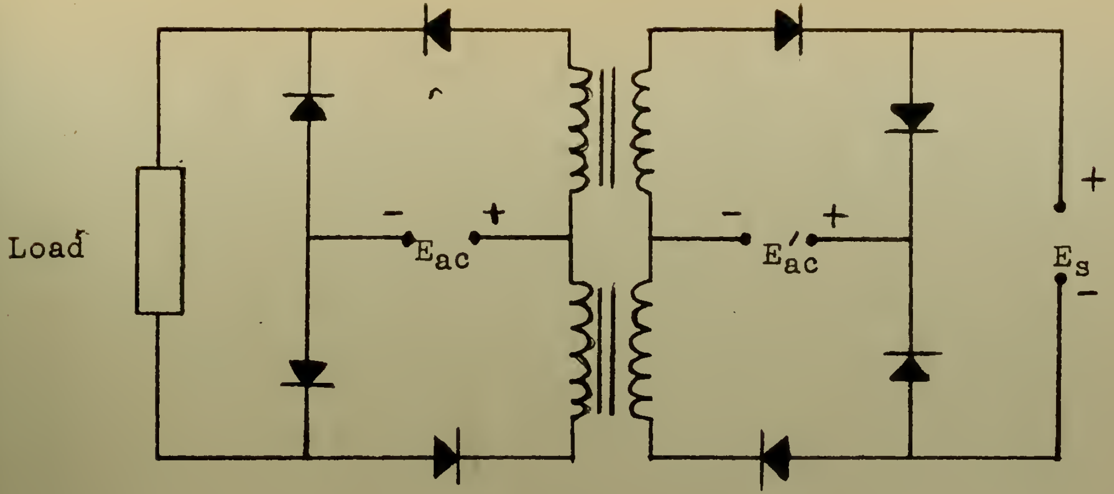

Figure 3 Single Core M/As in Push-Pull for Full Wave Output

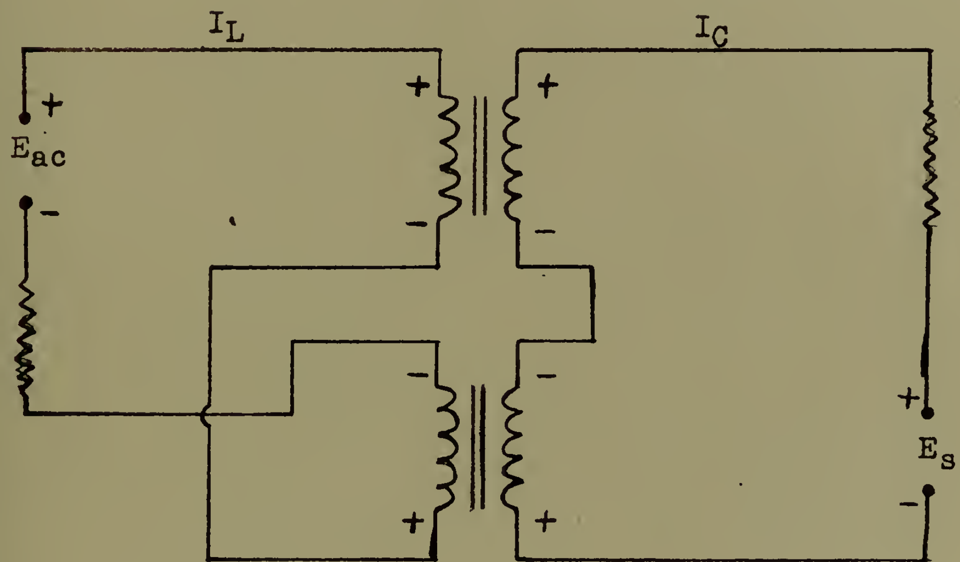

Figure 4 Series M/A Circuit

single core M/A gets its name. Another feature differentiating this type from most other M/A is that ac power is applied to the control circuit as well as the load circuit. The reasons for these features and the advantages gained thereby will be discussed presently.

2. Assumptions used in the mathematical treatment.

In order to explain the operation of the fundamental single core circuit in simple mathematical form certain assumptions must be made. The quality of materials now available makes these assumptions very close to actual conditions in some cases. If conditions in the operating circuit vary considerably from those assumed the resulting output will of course vary from that computed. The assumptions are as follows:

1. "Ideal" magnetization curves for the core materials, such as that of Figure 5.

2. "Ideal" rectifiers, ie. zero forward resistance and infinite back resistance.

3. Sinusoidal voltage sources.

4. Zero winding resistances and resistive load impedance.

5. Signal voltage, when present, is a rectified sinusoid in phase with the ac line voltage.

The operation of any M/A can be divided into two periods, during each of which the action may be represented by a different set of equations. These are (1) a period during which the output current is large, which we will call the conduction period,

and (2) a period during which the output current is very small (of the order of magnetizing current in actual circuits, zero in our assumed circuit) which we will call the non-conducting period. For the single core M/A (redrawn in Figure 6 for easy reference) the conduction period begins when the core reaches saturation, and the non-conduction period begins when the instantaneous value of the applied voltage E_{ac} drops to zero. This follows from our assumption of ideal core materials. Before saturation we are operating somewhere between "a" and "b" of Figure 5 where the slope of the magnetization curve is vertical. The winding offers a theoretically infinite impedance and the current must remain at zero value until a sufficient number of volt-seconds have been applied to fully magnetize the core, ie. bring its flux to the maximum or saturation level. Actually the current in the winding builds up to the magnetizing current value during the application of this voltage, but with the good core materials available, as described in the introduction, this is very small in comparison with the current which flows after the core reaches saturation. Of course, at the end of a half cycle when the applied voltage becomes zero again, current flow ceases even though the core is still saturated and the only impedance is the resistance in the circuit. To depart from our idealization once more, the flux does not actually remain exactly at the saturation level as shown in Figure 5 and assumed in our mathematical development. Instead it drops slightly from ϕ_s to ϕ_r as current flow and hence mmf decreases, as

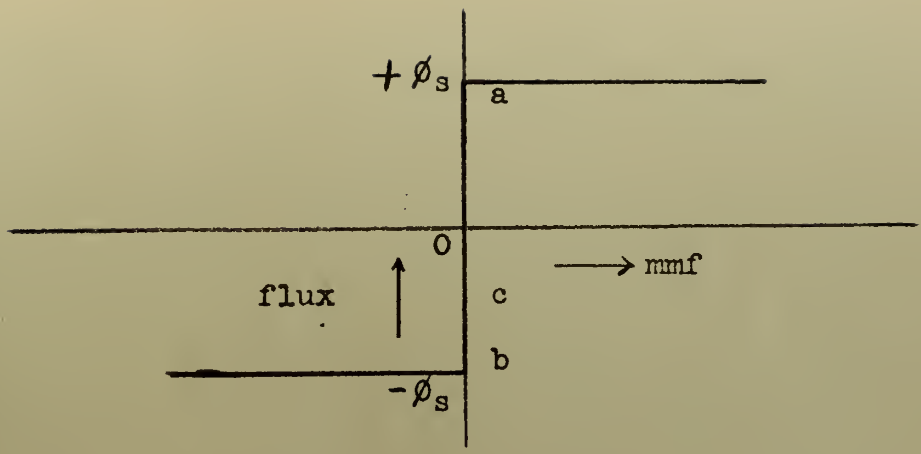

Figure 5   Ideal Magnetization Curve

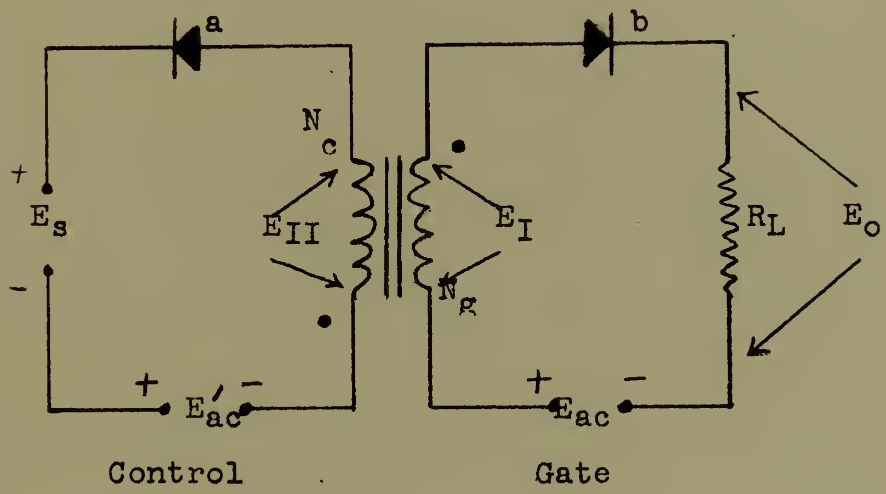

Figure 6   Single Core M/A

shown in Figure 1. However, for the modern core materials such as Deltamax this drop in flux density is very small since the remanent flux ϕ_r (the flux remaining when the mmf is returned to zero after bringing the core to saturation) is 98% of the saturation flux ϕ_m [4].

3. Voltage equations for conduction and non-conduction periods.

We will call the half cycles of the supply voltage during which polarities are as shown on the diagram positive half cycles, and the half cycles during which polarities are opposite to those shown will be referred to as negative half cycles. Because of the gate circuit rectifier, "b", a negative half cycle is always a non-conduction period. In the control circuit, however, the ac supply voltage is in the direction of zero rectifier resistance, so, referring to Figure 6, we have:

$$e'_{ac} = e_{II} + e_s \tag{1}$$

Any portion of the positive half cycle during which the core is not saturated is also part of the non-conducting period. e_{ac} is in the direction of zero resistance for rectifier "b" so:

$$e_{ac} = e_I \tag{2}$$

Since no current is flowing there is no voltage across R_L. In the control circuit e'_{ac} and the voltage induced by the gate winding $\frac{N_c}{N_g} e_I$ are both blocked by rectifier "a".

During the conduction period, which is the portion of the positive half cycle that the core is saturated and has no voltage across it, the voltage drop is entirely across the load, so:

$$i_L = \frac{e_{ac}}{R_L} \tag{3}$$

4. No-signal operation.

A starting point must be chosen in order to evaluate the definite integrals which are involved in our problem. Let us take the no-signal condition ($e_s = 0$) sufficiently long after application of the ac voltage so that transients due to its application have disappeared. The magnitude of E_{ac} is just sufficient to cause saturation of the core. This means the flux in the core is swinging from knee of the magnetization curve, "a", to "b" on Figure 5, but is lagging 90° in time behind the ac voltage, because the maximum positive flux occurs at the end of the positive half cycle of voltage. This is the point at which the time integral of voltage applied to the core reaches its positive maximum. Time relationships of power supply voltage, voltage applied to the load winding, voltage applied to the control winding, flux in the core, signal voltage, and load current are shown in Figure 7. The value of the output current is zero because the full value of volt-seconds applied to the core during each half cycle is just enough to change the flux from saturation in one direction to saturation in the other. Because of the rectifiers the mmf which changes the flux in one direction is supplied only by the load winding while the mmf which results in a flux change in the other direction is supplied by the control winding. There are different numbers of turns in each of these windings, so the voltage applied to each must be proportional to the number of turns. This is what determines the required ratio

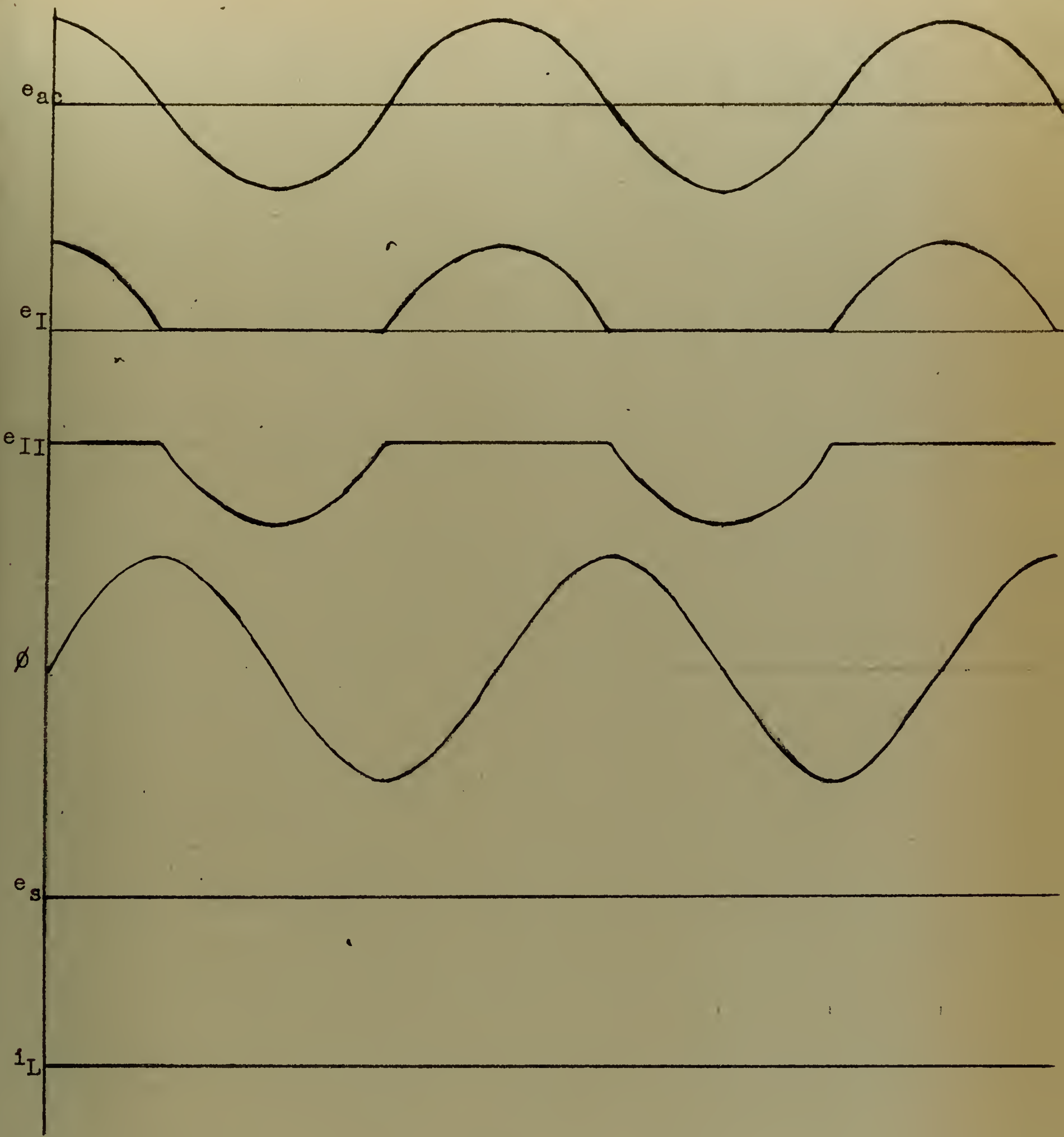

Figure 7 Voltage and Current Waveforms for the Single Core M/A in the No-signal Condition ($e_s = 0$)

of E_{ac} to E'_{ac} as mentioned on page 9.

5. Flux equations at time of signal application.

Now we are ready to apply a signal voltage e_s which is rectified ac in phase with e_{ac}. Let us begin application of this voltage at point "a" on the magnetization loop of Figure 5. This is the point at which e_{ac} is zero, $\frac{de_{ac}}{dt}$ is negative, and is the beginning of a negative half cycle. We will refer to this, and to the beginning of each succeeding half cycle as zero time. Each half cycle represents an elapsed time of $\frac{1}{2f}$ seconds, and since we are shifting our time reference each half cycle, time will have reached a value of $\frac{1}{2f}$ seconds at the end of each one.

During the first half cycle of ac voltage following the application of e_s, the core, which is positively saturated as this negative half cycle begins, is deviated from saturation an amount:

$$\Delta_1 = \int_0^{\frac{1}{2f}} e_{II}\, dt \tag{4}$$

e_{II} in this relation is obtained from equation (1), and is equal to:

$$e_{II} = e'_{ac} - e_s \tag{5}$$

Since e_s now has a finite value equal to $\frac{e'_{ac}}{2}$ in the example shown in Figure 8, the magnitude of e_{II} is reduced and its integral over the half cycle is less than the number of volt-seconds required to change the flux in the core from positive saturation to negative saturation. The excursion of the flux is now as shown in Figure 8 and it ends the first half cycle

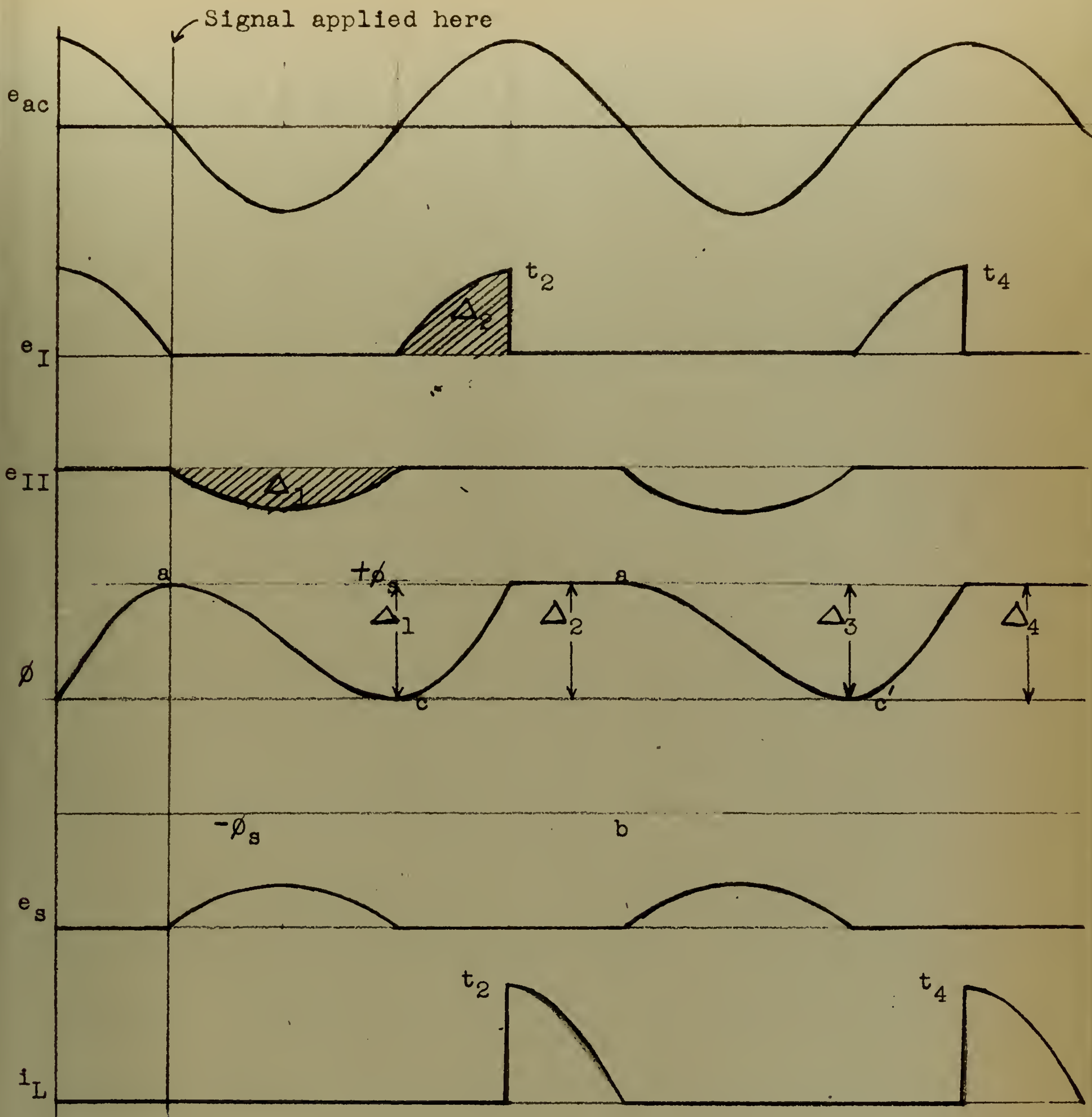

Figure 8 Voltage and Current Waveforms for the Single Core M/A with signal applied ($e_s = \frac{e_{ac}}{2}$)

at flux level "c" (shown also on Figure 5) instead of at "b".

During the second half cycle equation (2) applies. This condition will remain in effect until the core reaches saturation, which will occur at a time t_2 defined by the relation:

$$\int_0^{t_2} e_I \, dt = \int_0^{\frac{1}{2f}} e_{ac} \, dt = \Delta_2 \qquad (6)$$

Change of flux from "c" up to the value of positive saturation at "a" during the half cycle is shown on ϕ curve of Figure 8.

From time t_2 to time $\frac{1}{2f}$ there occurs a period of conduction during which the voltage e_I is 0. This is shown on the e_I curve of Figure 8 and the resulting sudden rise in current or "firing" is shown on the i_L curve of the same Figure.

During the ensuing negative half cycle (3rd) the gate circuit voltage is blocked by the rectifier and the core is deviated from saturation, its condition at the end of the second half cycle, by the control circuit voltage. The amount of this deviation is:

$$\int_0^{\frac{1}{2f}} e_{II} \, dt = \Delta_3 \qquad (7)$$

This brings the core flux back to point "c " on Figure 8.

Again in the fourth half cycle, the core is unsaturated (nonconducting period) from time 0 to t_4 . This time of firing is determined by the volt-seconds necessary to bring the core back to saturation, so we have:

$$\int_0^{t_4} e_I \, dt = \Delta_4 = \Delta_3 \qquad (8)$$

This action is repeated in succeeding half cycles and it is seen that during any positive half cycle n (n must be even for a positive half cycle) the core saturates and fires at a time t_n such that:

$$\int_0^{t_n} e_I \, dt = \Delta_{n-1} = \int_0^{\frac{1}{2f}} e_{II} \, dt \qquad (9)$$

In otherwords the area under each lobe of the e_I curve must equal the area under the preceding lobe of the e_{II} curve. One pair of these equivalent areas is shown shaded on Figure 8. Figure 9 shows the same set of voltages and current when $e_s = e'_{ac}$. Here the equivalent areas of the e_I and e_{II} curves representing amount of flux change in the core have both become zero, indicating that no reset takes place during the reset half cycle. Consequently the core is saturated even at the beginning of the gating half cycle and the full value of e_{ac} is applied to the load.

6. Flux equations for steady state operation

The actual value of Δ_{n-1} is a function of the voltage e_{II} applied during the (n-1)th half cycle. Since voltage is applied to the load circuit only during alternate half cycles, the condition for steady state operation after application of a control voltage of constant magnitude is:

$$t_n = t_{n-2} \quad \text{or} \qquad (10)$$

$$\int_0^{t_n} e_I \, dt = \int_0^{t_{n-2}} e_I \, dt \, .$$

From equation (9) it is seen that this may be written:

$$\int_0^{\frac{1}{2f}} e_{II} \, dt (\text{nth half cycle}) = \int_0^{\frac{1}{2f}} e_{II} \, dt \begin{bmatrix} \text{for (n-2)th} \\ \text{half cycle} \end{bmatrix} \qquad (11)$$

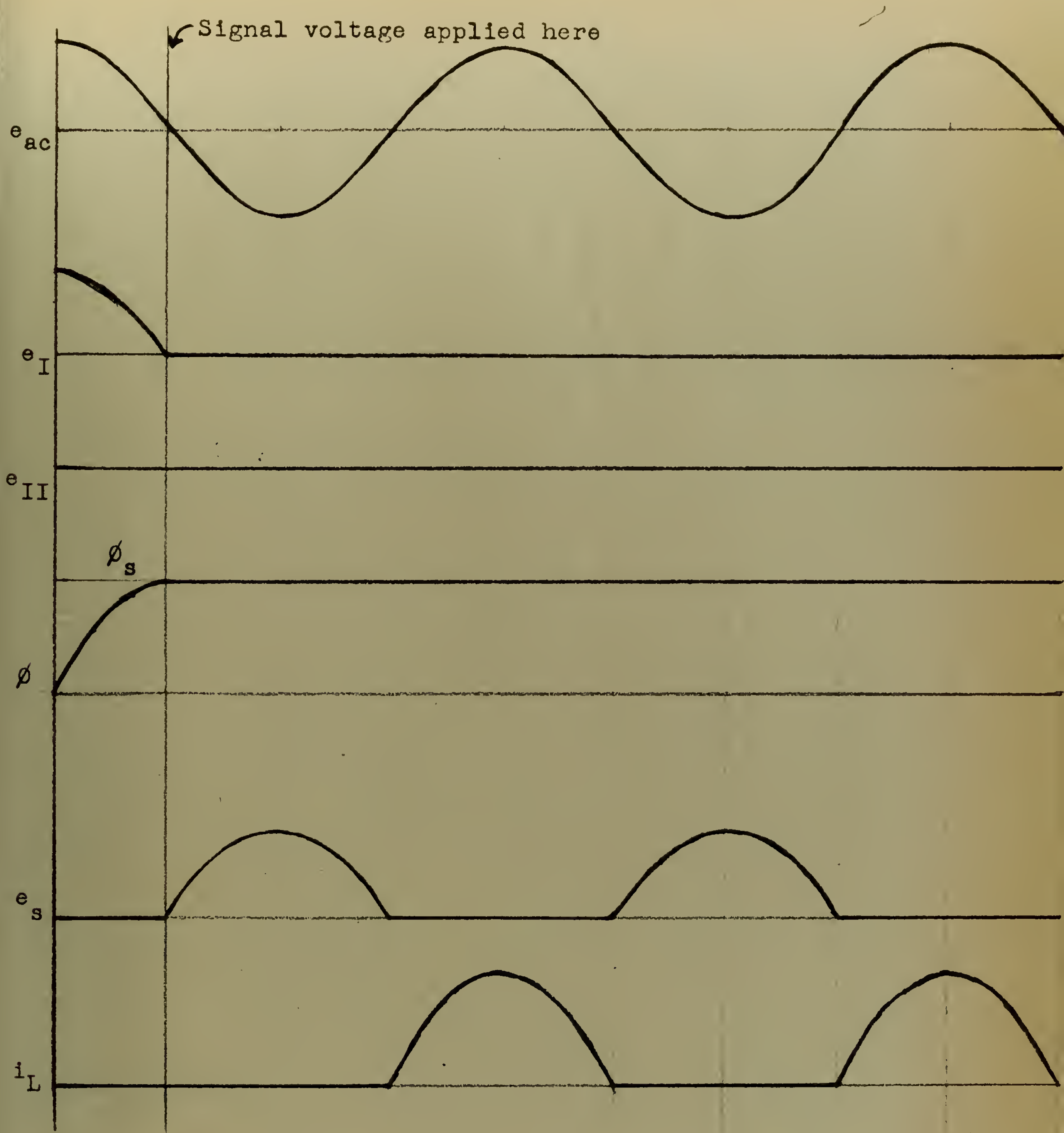

Figure 9 Voltage and Current Waveforms for the Single Core M/A with Full Signal Applied ($e_s = e'_{ac}$)

Substituting for e_{II} from equation (5) we have:

$$\int_0^{\frac{1}{2f}} (e'_{ac}-e_s)dt(\text{nth half cycle}) = \int_0^{\frac{1}{2f}} (e'_{ac}-e_s)dt\left[(n-2)\text{th half cycle}\right] \qquad (12)$$

We know that e_{ac} does not change from cycle to cycle so if e_s is the same in both the nth and (n-2)th half cycle we have steady state operation. This means that the output of the M/A which is determined by the firing angle, and is a direct function of t_n, reaches its steady state value within a cycle of the power supply frequency after the application of any new value of control voltage.

To evaluate t_n use equation (9) and substitute values from equations (1) and (2) for e_I and e_{II}. Expressing the voltages as functions of time t ($e = E \sin\omega t$) gives:

$$E_{ac}\int_0^{t_n} \sin\omega t\, dt = \left[E_{ac} - \frac{E_s}{N}\right]\int_0^{\frac{1}{2f}} \sin\omega t\, dt \qquad (13)$$

The value $\left[E_{ac} - \frac{E_s}{N}\right]\sin\omega t$ for e_{II} is obtained from equation (1) by expressing in terms of E_{ac} so that a given number of volt-seconds in either winding will have an equivalent effect on the flux in the core.

Integrating we have:

$$E_{ac}\left[\frac{-1}{\omega}\cos\omega t\right]_0^{t_n} = \left[E_{ac} - \frac{E_s}{N}\right]\left[\frac{-1}{\omega}\cos\omega t\right]_0^{\frac{1}{2f}}$$

Substituting limits gives:

$$E_{ac}\left[\frac{-1}{\omega}\cos\omega t_n + \frac{1}{\omega}\right] = \left[E_{ac} - \frac{E_s}{N}\right]\left[\frac{-1}{\omega}\cos\frac{\omega}{2f} + \frac{1}{\omega}\right]$$

Multiplying by ω and removing parentheses gives:

$$E_{ac} - E_{ac}\cos\omega t_n = E_{ac} - E_{ac}\cos\frac{\omega}{2f} + \frac{E_s}{N}\cos\frac{\omega}{2f} - \frac{E_s}{N}$$

Noting that $\cos\frac{\omega}{2f} = \cos\frac{2\pi f}{2f} = \cos\pi = -1$ we have

$$-E_{ac}\cos\omega t_n = -\frac{2E_s}{N} + E_{ac}$$

From this:

$$\cos \omega t_n = \frac{2E_s}{NE_{ac}} = \cos \theta_n \tag{14}$$

Since E_{ac} and N are constants, the firing angle is directly determined by E_s, and as we have seen above, it is the value of E_s during the preceding half cycle which controls the firing time of a given conducting half cycle.

7. Output current, voltage, and gain.

Now let us examine the output current. Its instantaneous value is given by equation (3). Since there is only one current pulse per cycle, the current must be averaged over the time of an entire cycle, $\frac{1}{f}$.

$$I_L \text{ ave} = \frac{1}{1/f}\int_{t_n}^{\frac{1}{2f}} i_L = f\frac{E_{ac}}{R_L}\int_{t_n}^{\frac{1}{2f}} \sin \omega t \, dt$$

Integrating, we have:

$$I_L \text{ ave} = f\frac{E_{ac}}{R_L}\left[\frac{-1}{\omega}\cos \omega t\right]_{t_n}^{\frac{1}{2f}}$$

After substituting limits:

$$I_L \text{ ave} = f\frac{E_{ac}}{R_L}\left[\frac{-1}{\omega}\cos\frac{\omega}{2f} + \frac{1}{\omega}\cos \omega t_n\right] \tag{15}$$

$$= f\frac{E_{ac}}{R_L}\left[\frac{1}{2\pi f}\cos \omega t_n + \frac{1}{2\pi f}\right]$$

$$= \frac{E_{ac}}{2\pi R_L}\left[\cos \theta_n + 1\right]$$

Substituting the value of $\cos \theta_n$ from equation (14)

$$I_L \text{ ave} = \frac{E_{ac}}{2\pi R_L}$$

Since E_s ave $\frac{1}{\pi}$ E_s this may be expressed as:

$$I_L \text{ ave} = \frac{E_s \text{ ave}}{NR_L} \tag{16}$$

Average output voltage is of course:

$$E_o \text{ ave} = I_L \text{ ave } R_L = \frac{E_s \text{ ave}}{N}$$

Voltage gain then is:

$$\left|\frac{dE_o \text{ave}}{dE_s \text{ave}}\right| = \frac{1}{N} = \frac{N_g}{N_c} \tag{17}$$

Under the assumptions made there is no control current and hence no power input. For this reason, power and current gains cannot be computed from these results.

8. Summary of operation.

The operation of the single core M/A may be summarized as follows: At the end of a gating half cycle the core is always saturated, having just completed a conducting period. The volt-seconds applied during the reset half cycle remove the core from saturation by an amount determined only by the relative value of e'_{ac} and e_s, since e_{ac} is blocked by the rectifier during this half cycle. If e_s is equal in magnitude to e'_{ac} there is no reset; if e_s is zero the core is completely reset since the full value of e'_{ac} acts on it throughout the half cycle.

During the gating half cycle e'_{ac} and e_s are blocked by the rectifier and the core is affected only by e_{ac}. The time integral of this voltage over the half cycle is just sufficient to saturate the core if it has been completely reset by e'_{ac}. Therefore if e_s was zero the previous half cycle, allowing complete reset, the core will be returned to saturation at the end of this half cycle. If e_s was not zero the core will reach saturation while there remains to be applied a number of volt-seconds of e_{ac} equivalent to the time integral of e_s during the previous half cycle. Since at saturation the voltage across the gate winding becomes zero, these volt-seconds will be applied to the load R_L. As with other M/A the output waveform is not a reproduction of the input. Only when E_s is so large

as to fully block reset of the core by e'_{ac} during the reset half cycle is the output in the form of a true half sine wave. This was shown in Figure 9 and is the condition in which the circuit is operated when used as a timer or delay line. At intermediate signal values its shape is generally that of Figure 8. However, its average value is proportional to the average value of the input voltage.

9. Some properties and uses.

M/A using this circuit are capable of short response time, high gain, good linearity, wide range of outputs, near independence of supply voltage, and good output power to weight ratios. [16]. Some of these characteristics will be investigated by examination of a specific single core M/A. First, let us see how these properties make this circuit especially suitable for ring counters, flip-flops, coincidence counters, and other devices usually made up with vacuum tubes.

The response time is not only short, but is of a definite value (1/2 cycle). The output does not rise exponentially as in most M/A, therefore in uses such as computers where a 100% change in state is desired the single core type can accomplish this in a half cycle and still show plenty of gain whereas other magnetic circuits with good power gain would require from three to six cycles [16] to approach full output. The simplicity of the single core circuit also makes it easy to combine several stages or several inputs to one stage if necessary to adapt it to various uses.

Examples [16] are shown in Figures 10,11, and 12. The coincidence counter of Figure 10 is a standard single core M/A with one or more extra control elements which are placed in parallel with the original control element. A voltage must appear at all the inputs in order to prevent reset and this influence the output of the stage. The ring counter or magnetic delay line of Figure 11 consists of cascaded single core M/A stages. The load impedance of one stage is the control impedance of the next stage. To complete the ring the output of the last stage is coupled back to the input of the first. Without this coupling the same circuit may be used as a straight delay line. The flip-flop of Figure 12 is a special case of the ring counter using only two stages and ordinarily having an arrangement so that the stages do not fire alternately, but change over from one to the other occurs only on receipt of an input signal—in other words, it has two stable states.

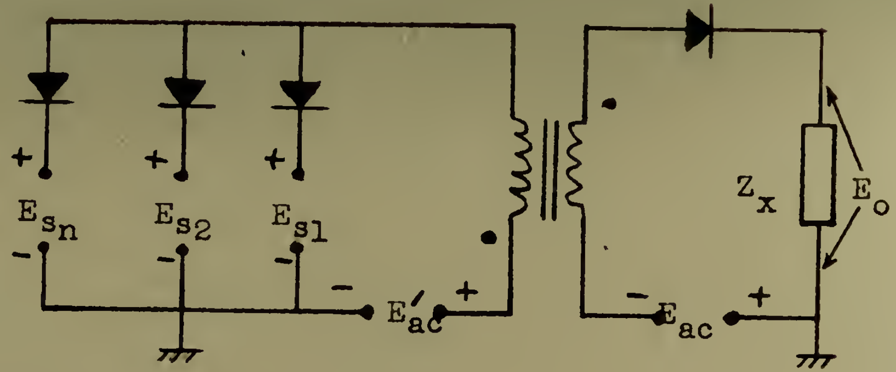

Figure 10 Coincidence Counter

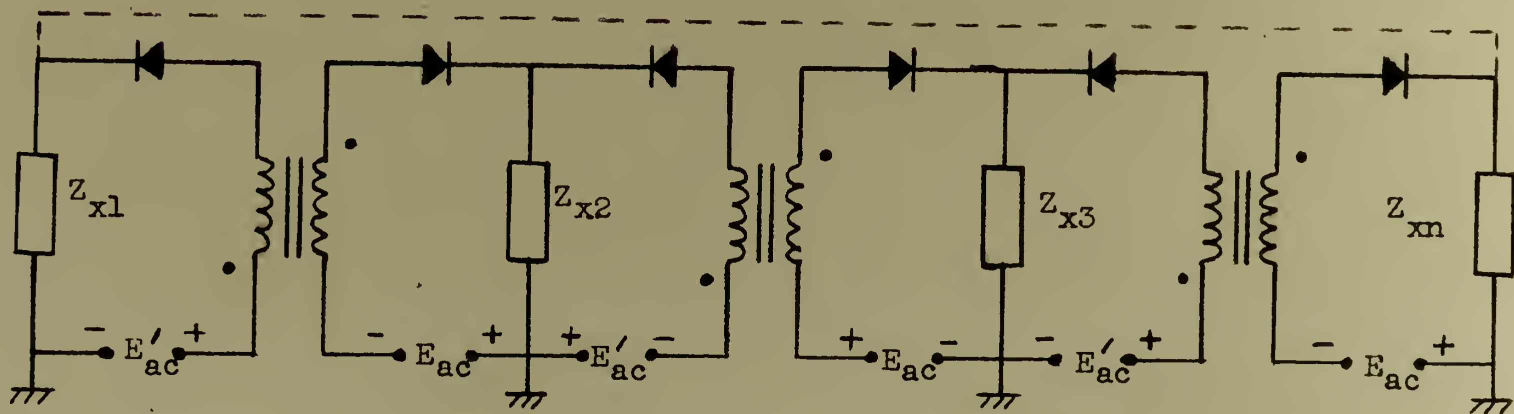

Figure 11 Ring Counter

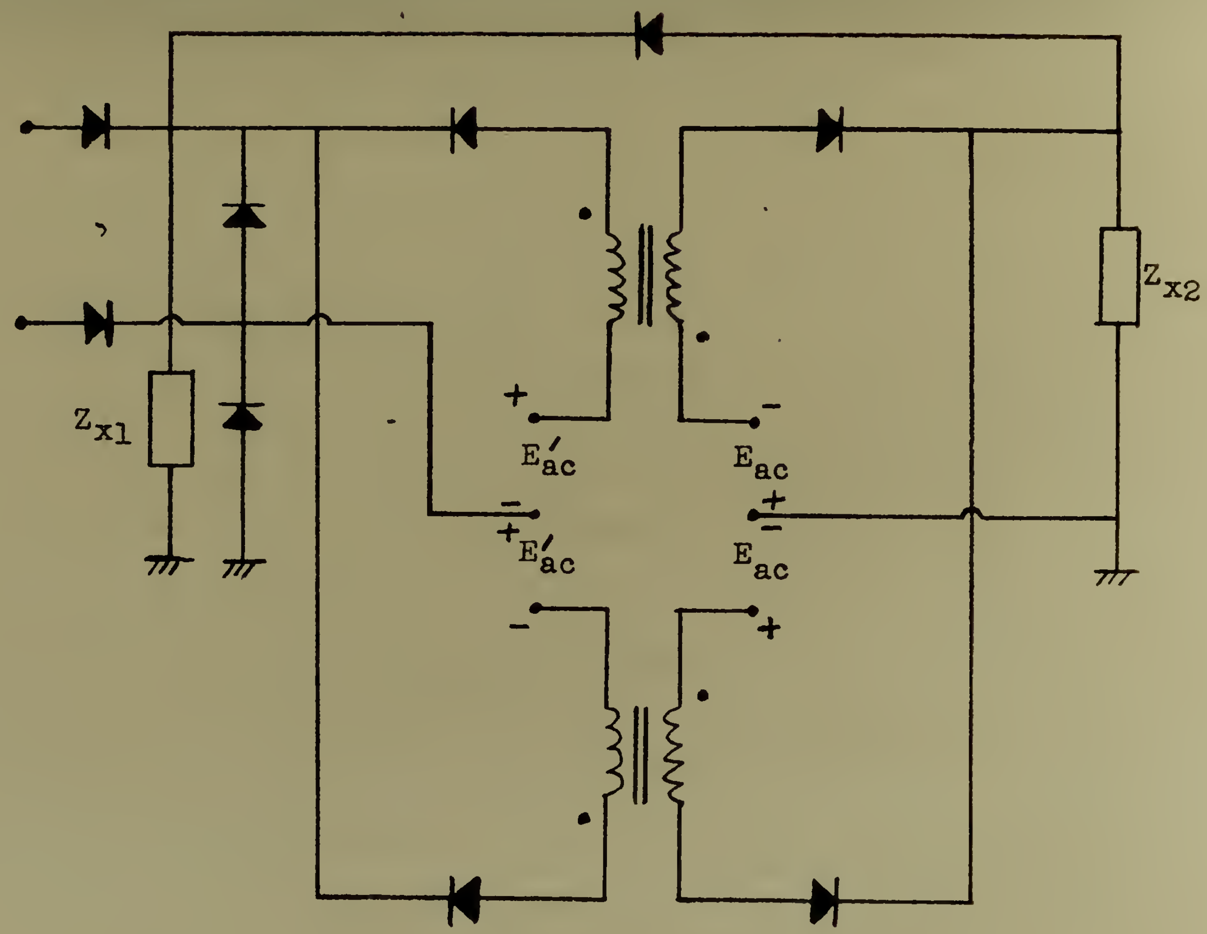

Figure 12 Magnetic Flip Flop

CHAPTER III

DESIGN OF M/A FOR MAGNETIC TIMER

1. General Considerations.

The author has used the single core M/A as the basic unit in a magnetic timer to give sequential operation of a series of 12 relays in an analog to digital converter. The relays were required to operate in a given order and at intervals of one cycle of the 400 cycle per second ac power supply. Relays used were Millisec Type B11[1]. They were chosen principally for their fast pull-in time, since they must operate during an output pulse which is at most a half cycle in length (1.25 milliseconds at this frequency). To insure the required rapidity of operation a voltage several times the nominal operating voltage of the relay was chosen as the saturation voltage of the M/A. The current required for relay operation serves as a guide for minimum wire size.

Although recently a refinement of the Ramey circuit has been described which allows most of its advantages to be attained with lower quality cores rectifiers [7], the effective operation of a single core circuit of the type described herein depend on "square loop" core properties so 50% Nickel - 50% Iron grain-oriented alloy was chosen. To take maximum advantage of the properties of the material by eliminating even a small air gap, tape wound toroidal cores are used. This configuration has been found worthwhile in most M/A using high

1. Manufactured by Stevens-Arnold Corp., South Boston, Mass.

permeability cores in spite of the fact that it requires special winding machines and causes less efficient use of the "window area" of the core because space must be left for the winder bobbin. In this instance Arnold Engineering Company's Deltamax cores were used. By reference to the core characteristic table (published by the core manufacturer) the number of turns per volt and the window area available for windings were determined. Maximum voltage output desired multiplied by volts per turn for a given core gives the number of load winding turns. The ideal theoretical voltage gain is equal to the turns ratio, as we have seen. Since in the counter the output voltage of one stage will be the input voltage of the next it is necessary to use somewhat better than a 1:1 ratio so that each core can be fully reset by the previous one in spite of losses. 1.5 to 1 was chosen and this determined the number of control winding turns. Since these were experimental models the same wire size was used in both windings for convenience. A core size was chosen large enough to allow room for both at a 30% space utilization factor. More detailed design methods for single core M/A are given by L. J. Johnson [9].

2. Final specifications.

As a result of these considerations, the M/A on which the experimental data of the succeeding sections were taken consisted of an 800 turn control winding and a 1200 turn gate winding, both of #30 AWG Heavy Formvay coated cooper, wound on an AECO[1] type 4635-D2 core. The core is wound from a Deltamax

1. Arnold Engineering Company

strip two mils in thickness. It is one inch in inside diameter, one and three-eighths inches in outside diameter and one-quarter of an inch in height. It is silicone impregnated and enclosed in a nylon case. The rectifiers used with it were type 1N58A germanium rectifiers[1]. These were chosen because they are very small, light in weight, glass-sealed against moisture, and will withstand back voltages up to 100 volts.

3. Testing methods.

After the cores were wound their saturation voltages were checked carefully. Since a number of stages were to be operated from the same power supply it was essential that turns ratio and saturation voltage ratio be alike for all so that the same ratio of E_{ac} to E'_{ac} could be used throughout. The circuit used for measuring the saturation voltage of each winding was a very simple one and since it could also be used to measure the volts per turn ratio of a core with a few turns on it in case a table of core characteristics was not available, perhaps it will be worthwhile to describe it briefly. The circuit is shown in Figure 13 but the component values in the sketch are not critical.

The winding whose saturation voltage is to be determined is placed across terminals a and b and the ac voltage at terminals one and two is gradually increased by means of a Variac. The horizontal deflection of the oscilloscope trace is due to the voltage across the small resistor in series with the coil

1. Manufactured by Sylvania Electric Company

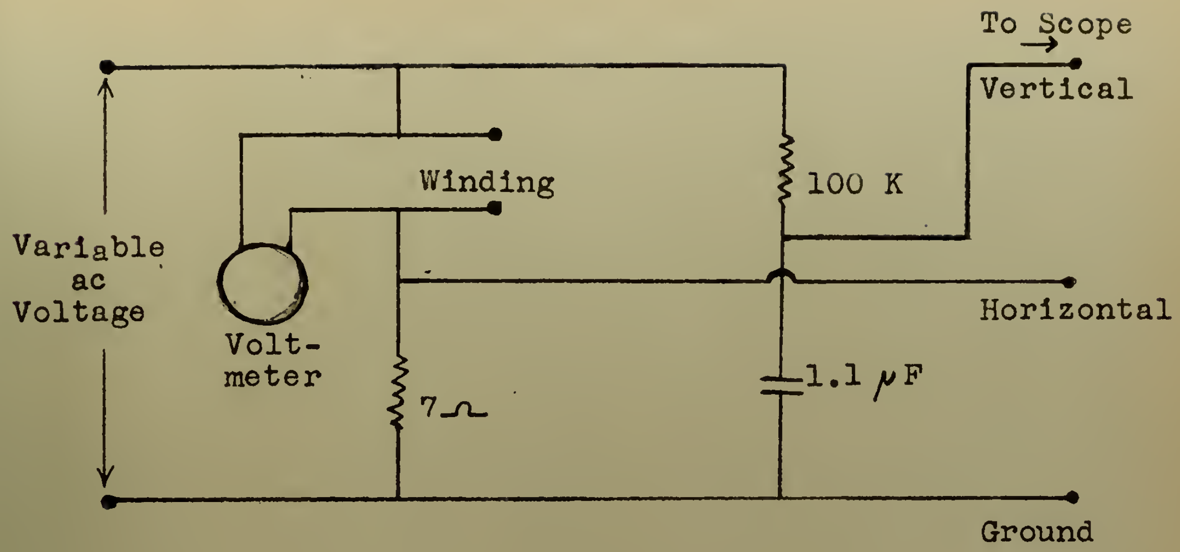

Figure 13   Schematic Diagram of Saturation Checker

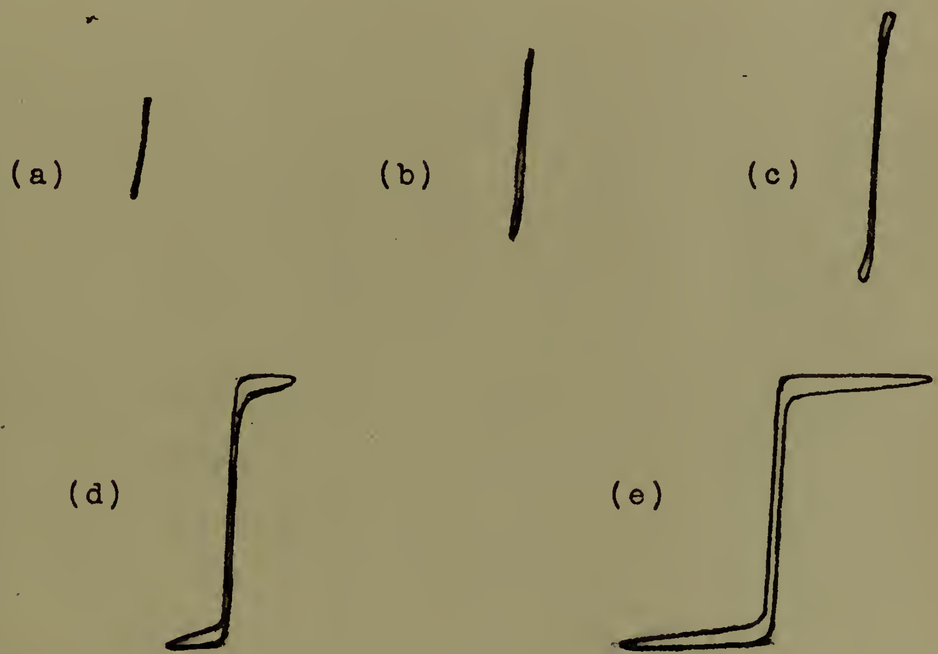

Figure 14   Waveforms as Voltage is Increased

and thus is proportional to the current through the coil. The vertical deflection of the trace is proportional to the voltage across the capacitor in the RC branch, which approximates the integral of the applied voltage. Since the flux density in the core is also determined by the time integral of the applied voltage, the oscilloscope displays a plot of flux density versus ampere turns, or the hysteresis loop of the core.

For Deltamax and the other "square loop" materials the waveforms seen by this method are as shown in Figure 14 (a) through 14 (e) in order of increasing ac voltage. The value of the applied voltage when the hysteresis loop just begins to reach the saturation point, Figure 14 (c), is the saturation voltage.

4. Theoretical versus experimental voltage gain.

As has been shown above, the ideal theoretical voltage gain is equal to the turns ratio. Miller and Hughes [8] have derived an equation for voltage gain which includes the effects of control circuit resistance and non ideal core characteristics. They assume a rectangular flux current loop of constant width, and using equations representing the conditions during each period somewhat as in the ideal development given above, they solve for the average output voltage (E_o ave) in terms of signal voltage (E_s), ac supply (E_{ac}), and other parameters of the circuit. Their results give gain as:

$$G = N\sqrt{1 - X^2} \quad \text{in which}$$

$$X = \frac{H_c \, L \, R_{ct}}{K \, N_c \left(1 - \frac{E_s}{E'_{ac}}\right)}$$

For the M/A used in the ring counter built by the author, the values of these parameters for two representative signal levels and the numerical solution for gain in each are given in Appendix A. The results are 1.495 at 7.07V E_s and 1.453 at 28.2V E_s. Curves of output current versus input voltage based on measurements in the laboratory are shown as Figure 13. Values of gain at the signal levels for which computations were made show the experimental voltage gain to be lower than expected from the Miller-Hughes equation. Variations of coercive force H_c from the nominal value for the material might cause minor differences in gain, but the chief variable in the circuit is the rectifier resistance.

The assumptions of Hughes and Miller include ideal rectifiers. However, they use the total control circuit resistance R_{ct}, which in this circuit must include the forward resistance of the rectifier. With the parameter values of this example the value of X is normally small so that even though it is proportional to R_{ct} it does not actually change the value of G by an appreciable amount unless E_s is very nearly as large as E_{ac}. However in the gate circuit the output voltage is measured across R_L which is in series with the gate circuit rectifier as well as with the gate winding and E_{ac}. The forward resistance of the rectifier is by no means inappreciable compared to R_L. It appears that the assumption of infinite back resistance for these rectifiers is a good approximation, but the assumption of zero forward resistance, though normally associated with the

ideal rectifier, does not lead to a useful value of gain for a circuit with this type rectifier. Rectifier resistance varies with current and this varies with E_{ac} during firing time of the M/A. Also there is some variation of resistance between individual rectifiers. The forward resistance of the 1N58A's used in this counter averaged 162Ω at 1 Volt applied voltage. If we could take the gain resulting from the formula and multiply it by a factor equal to the ratio of R_L to $[R_L + R_R]$ we would have a value much closer to the actual gain. However, this ratio is not a constant since the effective rectifier resistance changes with current. If the value measured as described above (162Ω) is used, the computed gain is .972 at 7V. and .944 at 28V. These values are much closer to the 1.07 and .835 which were measured at the same inputs than those resulting directly from the Miller-Hughes gain equation.

The above discussion will suffice to show the need for a turns ratio of at least 1.5 to realize a voltage gain of 1 across the stage. The characteristic curves (Figure 15) show the very good linearity which is readily achievable in this M/A. For counter use, however, linearity is unimportant and gain need only be at least one because the input signal will either be zero or large enough to completely block reset of the core. A gain of more than one will do no harm but less than one would mean a constantly decreasing signal amplitude.

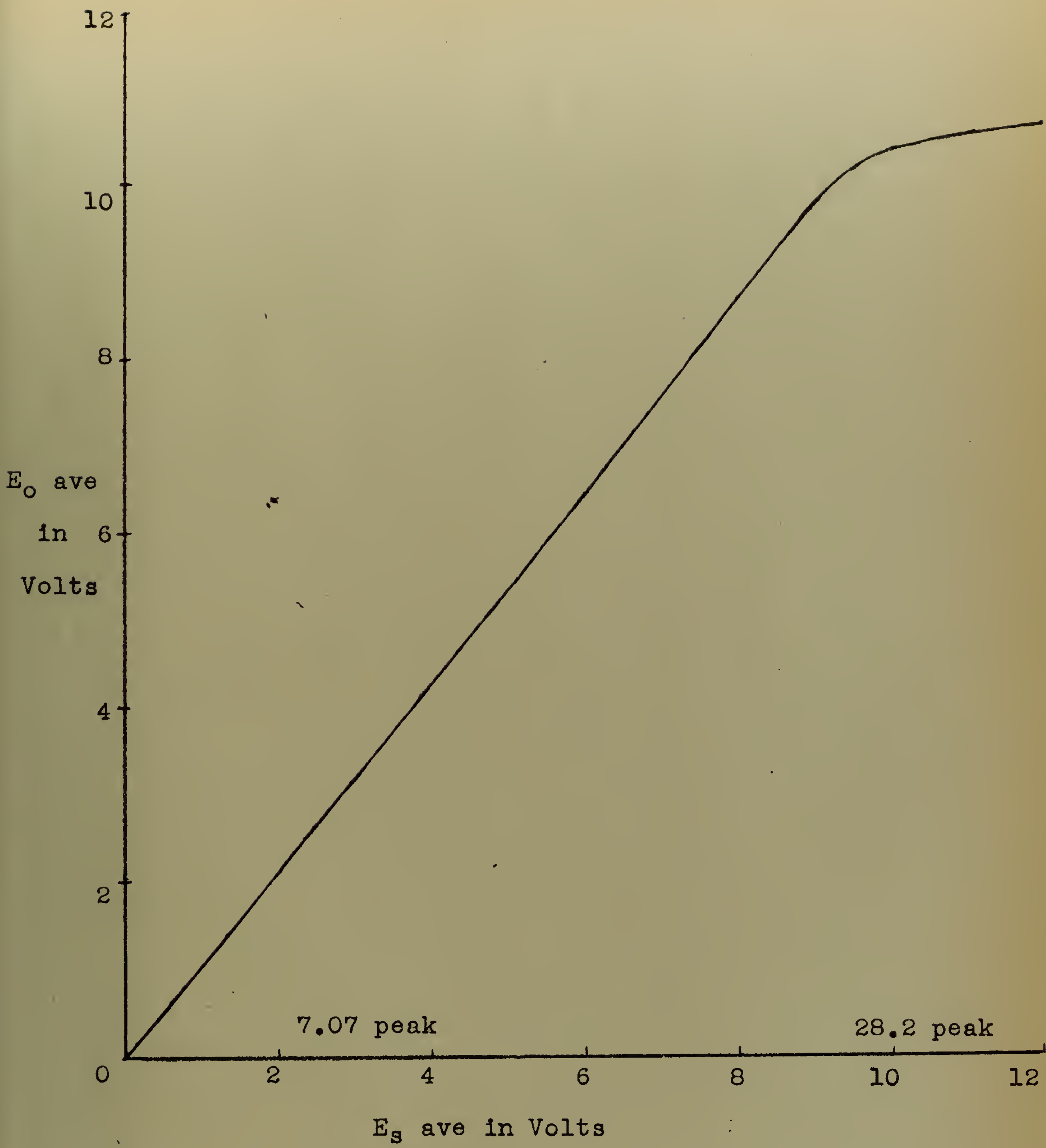

Figure 15 Input Voltage versus Output Voltage Curves for a Single Core M/A

CHAPTER IV

OPERATION OF MAGNETIC TIMER

1. Origin of delay in the timer.

When several of these M/A are used in cascade in the counter circuit of Figure 11, the purpose is not to achieve high gain, but to make use of the time delay inherent in each state to provide a delay line action or a timed series of pulses. The operation is as follows: A pulse at the input of stage one during a reset half cycle is reporduced at the output of stage one during the next half cycle. This provides a signal pulse for stage two and during the succeeding (third) half cycle is reproduced again at the output of stage two where it acts as the signal pulse for stage three. The pulse is then reproduced in the output of stage three during the fourth half cycle after the signal was originally applied, and so on. The pulse travels through the series of counter stages essentially unchanged except that it is delayed by a half cycle in each stage.

2. Setup for measuring delay.

The timer constructed by the author consisted of 24 stages in order to provide a total delay of 12 cycles. For experimental purposes an input pulse was provided by a multivibrator and cathode follower. A schematic diagram of the pulsing unit designed by the author for this purpose is shown as Figure 16. The multivibrator is the single shot type and is triggered by

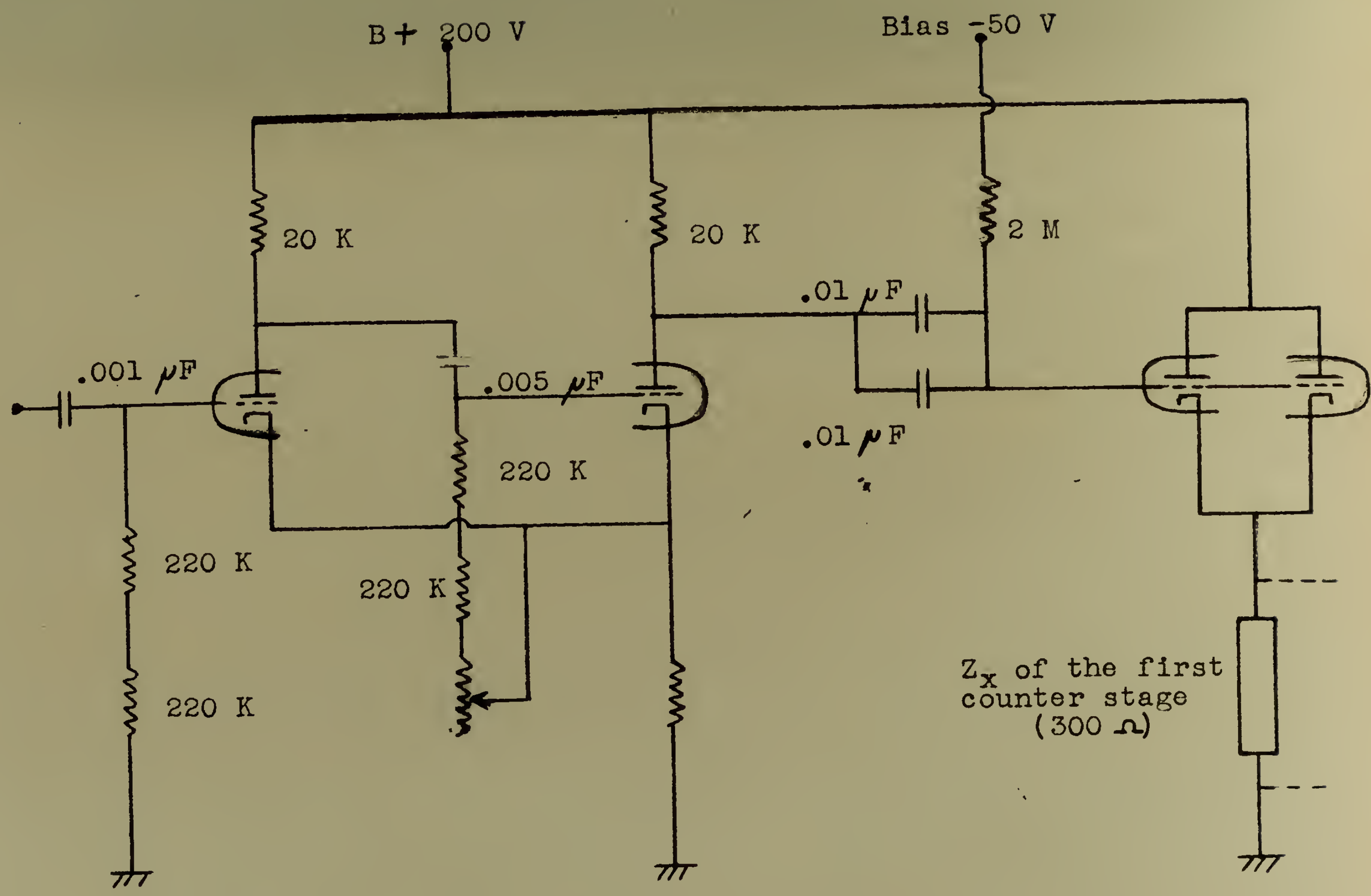

Figure 16 Pulse Generator for M/A Delay Line

Tubes are type 5687

a variable frequency square wave generator in order to afford a separately variable repetition rate independent of the pulse length adjustment. The square wave generator is synchronized with the power supply frequency so the pulse produced by the multivibrator will always be in the same time relationship to the reset cycle of the first counter stage. Representative wave forms at several stages of the counter showing their time sequence are sketched in Figure17. A block diagram of the equipment used and its interconnections is given in Figure 8. Theoscilloscope is a Tektronix type 535 with the 53C dual trace plug-in unit to facilitate measuring the time relationships between waveforms at various stages.

3. Operation of timer as a ring counter.

The same device may be operated as a ring counter by coupling the output of a given stage back to an earlier stage through a rectifier. Then a pulse originated by the multivibrator will circulate indefinitely after the multivibrator is disconnected. There must be an even number of stages in the ring in order to bring the last stage output pulse back to the first stage input during a reset half cycle. The megnetic counter was operated as a ring counter during experiments to determine the effects of varying circuit parameters on the theory that these variations would effect the circuit more with its operation entirely dependent onthe ability of each stage to reproduce a pulse sufficiently like its input to operate the next stage, instead of having a new signal pulse

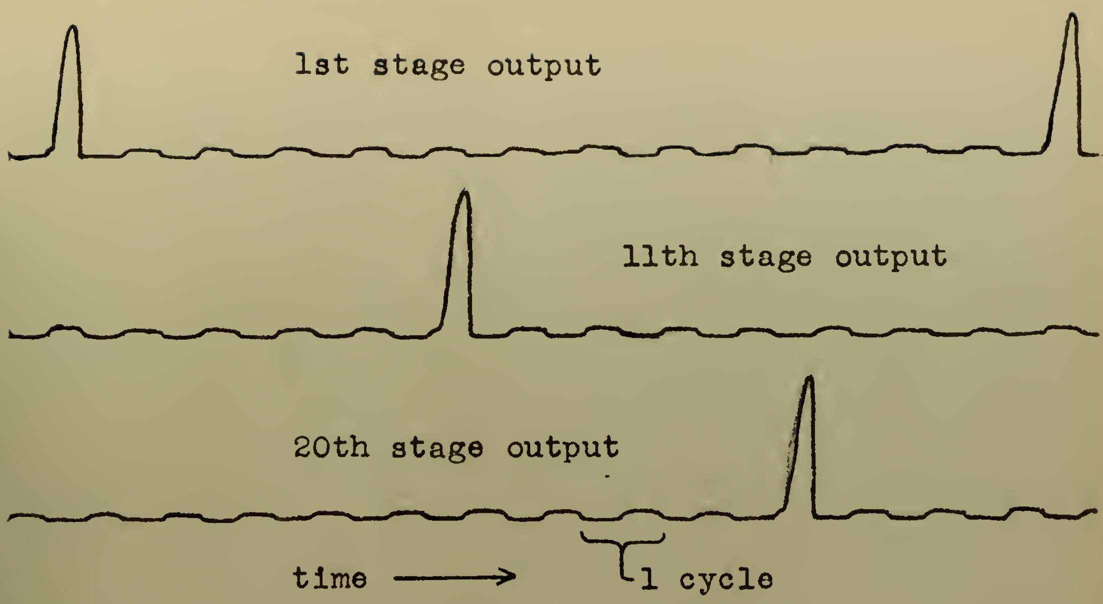

Figure 17 Time Relationship of Voltages in Various Stages of the Magnetic Delay Line

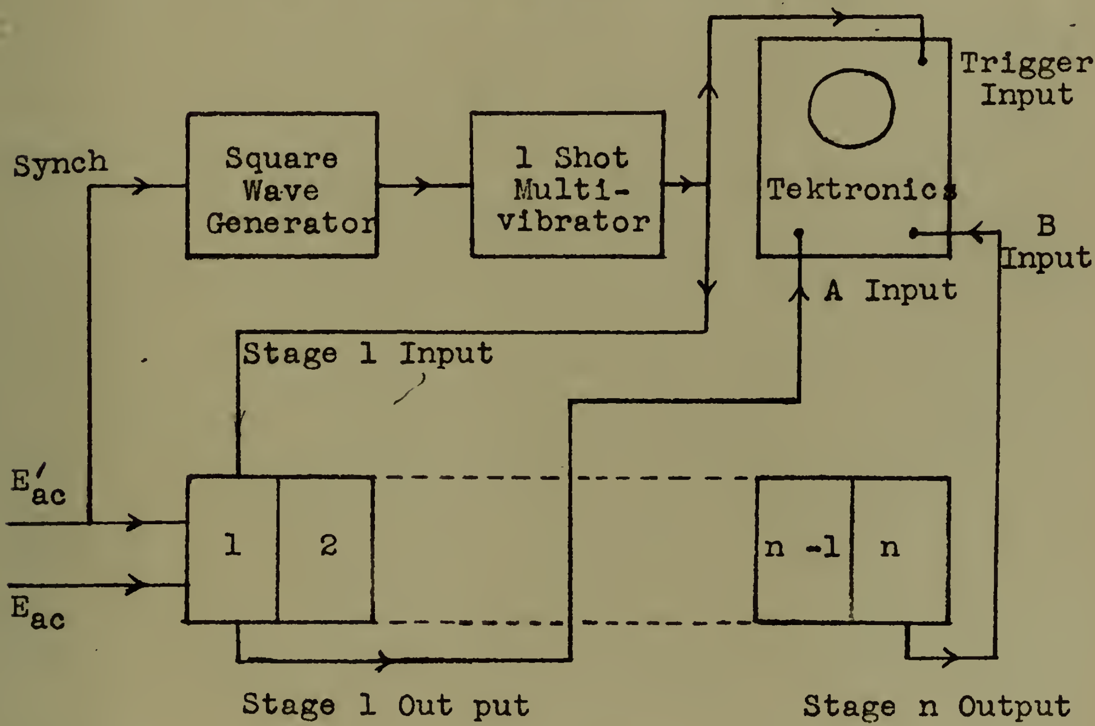

Figure 18 Equipment setup for Measuring Time Delay

inserted at the beginning of each traversal of the counter chain.

The output of stage 21 was connected to the input of stage two forming a circuit like that of Figure 19. After introducing a pulse with the multivibrator this pulse is repeated by each M/A stage in turn so that at any given stage a pulse will appear every 20 half cycles, or every 10 cycles of the supply frequency. At 400 cycles this means a pulse every 25 milliseconds. By varying the number of stages this time could be made equal to almost any multiple of 2.5 milliseconds with an accuracy equivalent to that of the power supply frequency.

4. Effects of circuit fluctuations on timer operation.

Tests on this ring counter showed the line voltage could be varied from approximately the saturation voltage of the cores, 80 V. rms, down to 30 V. rms with no change of pulse timing or other effect on the circuit, except a decrease in pulse amplitude as E_{ac} decreases. The coupling impedance (Z_x in Figure 19) is not critical either. The value in most stages during these tests was 150Ω, but it was found that any one impedance could be increased to 300Ω or decreased to 100Ω without interfering with the counting sequence. A counter of this type whose normal supply voltage was set to a value about 70% of saturation, and whose coupling impedances were of a median value, could be expected to operate without "losing count" through supply voltage variations of $\pm$30% and impedance changes in any stage of $\pm$50%.

CHAPTER V

RELAY OPERATION BY SINGLE CORE M/A

1. Requirements for single stage operation.

The experiments above have shown that the single core M/A can be used very satisfactorily in a counter circuit. How the counter can be used to time the closures of a set of relays remains to be seen. For least complexity and fewest components it was decided to operate the relays directly from the M/A output pulses if possible. This meant using the relay coil as part of the coupling impedance (Z_x, Figure 11 or 19) of every other stage in the counter. The reason they are needed only in alternate stages, of course, is that the relays were to operate at one cycle intervals while the delay in the counter is one-half cycle per stage.

As already mentioned, the relays were Millisec type B 11 to meet the short closing time requirement. Actually it was found that after a number of stages the signal pulse tends to decrease in width because any leakage in the gate circuit rectifier will tend to reset the core even though the reset voltage E'_{ac} is blocked by an incoming signal. The width of the pulse which must close the relay is less than a millisecond in some stages, although the full half cycle is 1.25 ms. However, the relay, though listed by the manufacturer as having a 1.0 ms. firm contact closing time when driven from a 6 V. constant current dc source, was found capable of firm contact in as

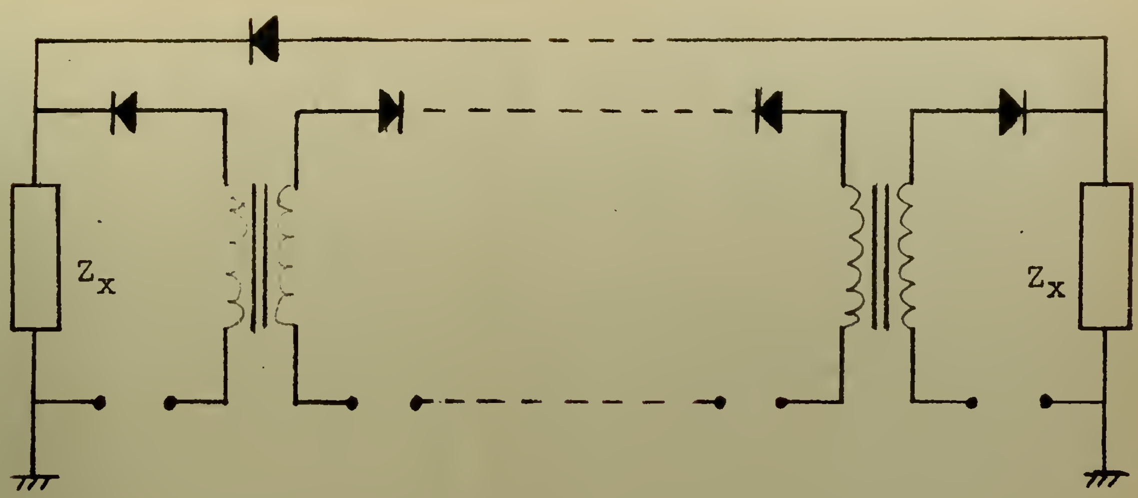

Figure 19 Magnetic Ring Counter

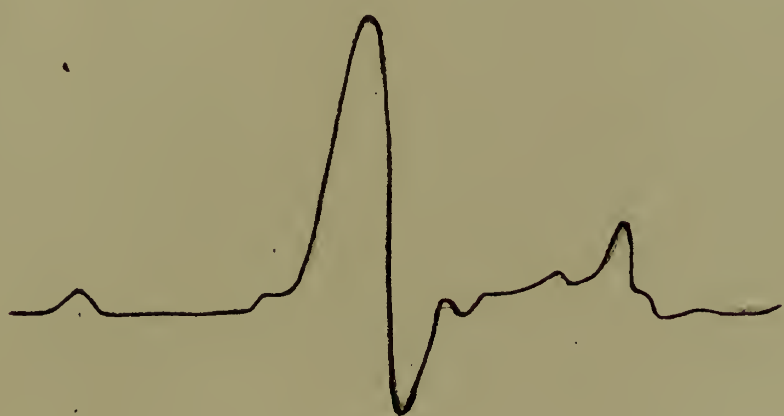

Figure 20 Voltage across Load including Uncompensated Relay

little as .3 ms when driven by the M/A output pulse whose peak amplitude was about 40 V. The M/A output pulses tend to have a steep leading edge which aids in getting fast closure time. The variations in shape of the top and trailing edge of the pulse did affect the amount of relay bounce or chatter as will be discussed shortly.

2. Requirements for multistage operation.

Eventhough the relay could be operated by the pulse from any stage, another problem developed when relays were inserted in all 12 stages. The reactance of the relay coil (about 1.2 millihenries) distorted the waveform of voltage across Z_x as shown in Figure 20. Since this voltage constitutes the input to the next stage, something must be done to restore it to a usable form which will trigger the succeeding stage properly. Otherwise the voltage pulse will become progressively more distorted as it passes through a series of counter stages. Several simple compensation methods were tried to reduce this effect. Circuits used and a sample of the results obtained in terms of voltage waveform and relay action are shown in Figure 21. Decade capacitors were used and values from 0 to 10 microfarads were tried. The rectifier was type 1N92. The compensating circuit requiring the fewest components was the capacitor shunting the relay coil. Since it gave results as satisfactory as those which could be dependably achieved with any of the others, it was chosen for use in the counter. The effect on the output waveform of varying the capacity across

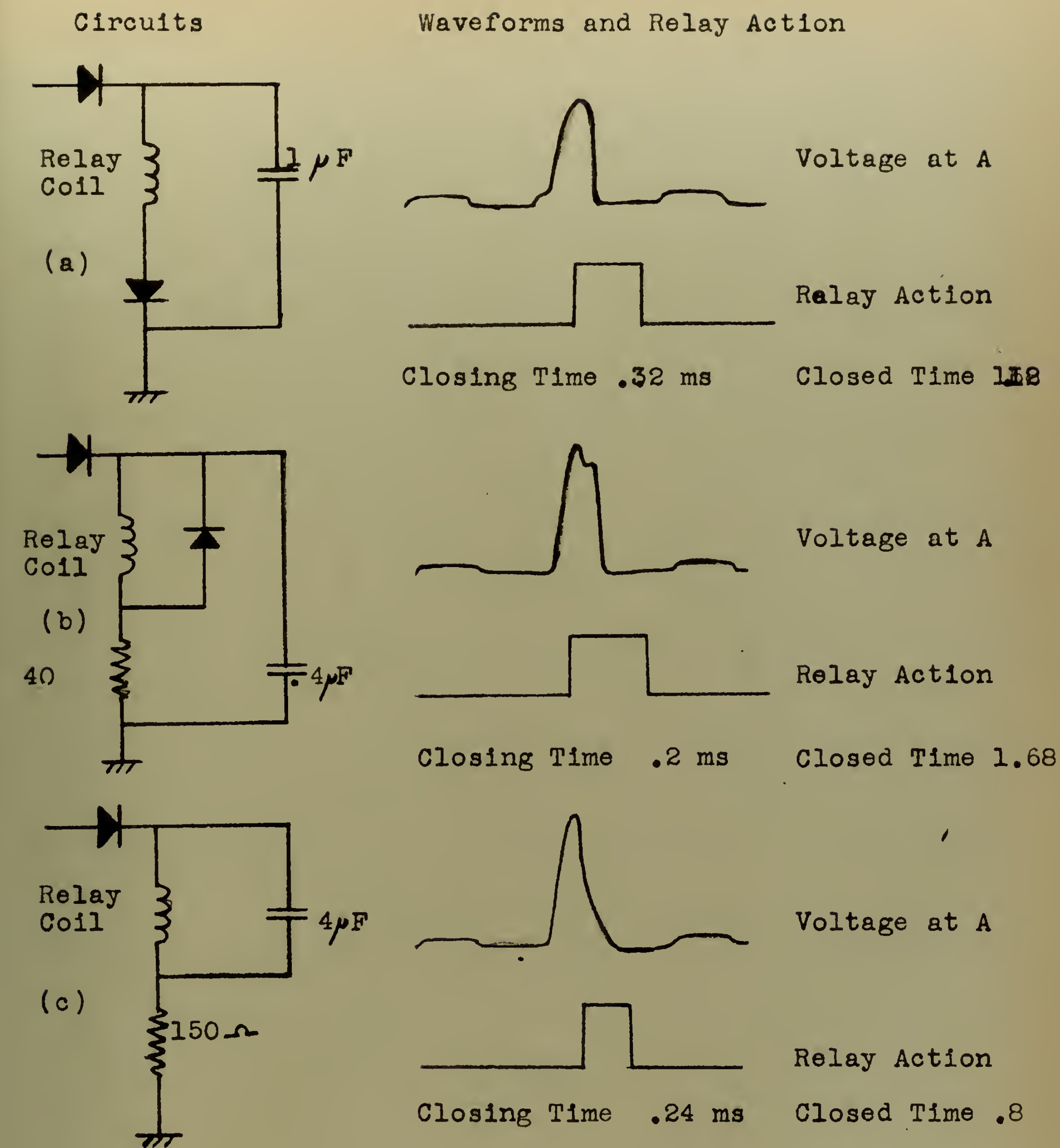

Figure 21 Some Compensating circuits and their Results

the relay coil is shown in Figure 22. The optimum value of capacitance turns out to be about 5μF for this circuit and this relay. It would appear from theory that the best results would be achieved when the relay coil was resonated by the capacitor to give an entirely resistive impedance as a load for the M/A. However, this can be done only at one frequency and the output voltage even in its ideal form is a half sine wave followed by a long period of zero voltage, which means it is the sum of an infinite series of frequencies. The capacity which gives best results is that which resonates with the coil at approximately the fifth harmonic of the power supply frequency. Apparently that harmonic is present in a disproportionate amount in the actual wave shape which, as can be seen in Figure 22, differs considerably from the theoretical half sine wave. At any rate the form of output pulse which gives proper operation of the next counter stage is close enough to that which gives proper operation of the next counter stage is close enough to that which gives non-chattering operation of the relay in this stage so that relay operation in every other stage of a 24 stage magnetic counter can be reliably obtained once the proper compensating circuit is determined.

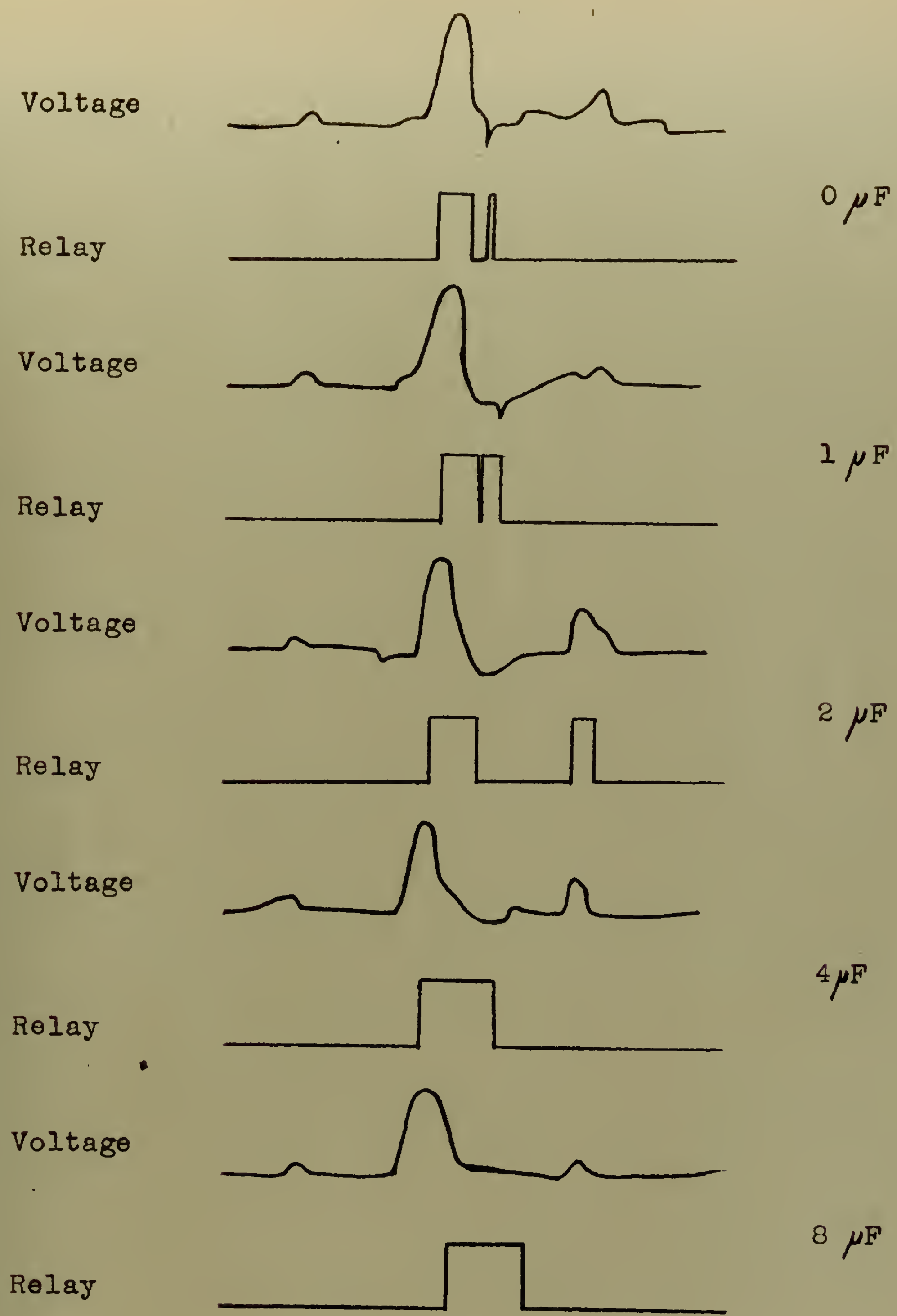

Figure 22 Waveforms and Relay Action with varying Compensation

CHAPTER VI

CONCLUSIONS

Results of the experimental work described above show that the delay line of single core M/A is adaptable to the timed operation of relays, giving a precise delay between successive closures quite independent of voltage fluctuations and circuit impedance changes. This is only one type of application, but it points the way to the practicability of the M/A for many computer and control purposes in which a timer, delay line, or nonmechanical rotating switch is desired. The very convenient means of combining this type of M/A into coincidence counters and flip-flop circuits as well as ring counters, using the same type of cores, windings and rectifiers for all stages make it expecially suitable for such uses. For the many uses not so dependent on the property of half cycle response the single core M/A has to compete with other types of saturable reactor devices as well as vacuum tubes, gas tubes, and so forth. It has proven adaptable to most of these [2, 5, 6, 10, 13, 18, 19].

In common with most devices the single core M/A has unfavorable characteristics as well as favorable ones. Principal disadvantages are: the input signal must be applied in phase with the power supply though this can be done in a variety of ways [16], ac power must be applied to both control and gate windings at a rather precise ratio of voltages,

satisfactory operation is dependent on good quality rectifiers, especially in the gate circuit, and again because of the rectifiers, which have a finite voltage drop, this circuit is not adapted for use with low level signals. These features must be considered before use of this circuit in solving a given design problem is made.

The advantages have been mentioned, but may be summarized briefly as follows: Fast, but definite, rather than exponential, response time available with good power gain, independence of supply voltage and circuit impedance due primarily to lack of bias windings, and the fact that core matching is unnecessary in its construction since the cores do not have to work in pairs. These features, together with the advantages common to most M/A such as reliability and low heat dissipation will lead to continuing increases in the use made of this circuit.

BIBLIOGRAPHY

1. Alexanderson, E. F. W. A MAGNETIC AMPLIFIER FOR RADIO-TELEPHONY
Proc. I.R.E., Vol. 4,
pp 101, Feb., 1916

2. Benjamin, F. IMPROVEMENTS EXTEND M/A APPLICATIONS
Electronics, pp 119, June, 1952

3. Davidson, J. D. APPLICATION OF M/A TO ELECTRONIC COMPUTERS
Thesis D_{165}, USNPGS, 1953

4. Ettinger, G. M. MAGNETIC AMPLIFIERS
Methuen's Monographs on Physical Subjects, pp 82, Methuen and Co., Ltd, London, 1953

5. Finzi, L. A. and Jackson, R. R. THE OPERATION OF M/A WITH VARIOUS TYPES OF LOAD
Comm. and Elect., No. 13, pp 270 and 279, July 1954

6. Finzi, L. A. and Mathias, R. A. AN APPLICATION OF M/A CIRCUITS TO PERFORM MULTIPLICATION AND OTHER ANALYTICAL OPERATIONS,
Comm. and Elect., No. 8, pp 455
Sept., 1953

7. House, C. B. FLUX PRESET HIGH SPEED M/A
Comm. and Elect., No. 10, pp 728
Jan. 1954

8. Hughes, G. E. and Miller, H. A. FAST RESPONSE M/A
Comm. and Elect., No. 11, pp 69
Mar., 1954

9. Johnson, L. J. HIGH SPEED M/A
Electrical Manufacturing, pp 98
Nov. 1952

10. Lord, H. W. M/A CIRCUITS WITH FULL WAVE OUTPUT AND HALF WAVE CONTROL SIGNALS
Comm. and Elect., No. 13, pp 265
July, 1954

11. Miles, J. G. — BIBLIOGRAPHY OF M/A DEVICES AND THE SATURABLE REACTOR ART
AIEE Technical Paper 51-388
Sept., 1951

12. Miles, J. G. — TYPES OF M/A - A SURVEY
AIEE Technical Paper 52-216
May, 1952

13. Ramey, R. A. — M/A CIRCUITS AND APPLICATIONS
Electrical Engineering, pp 791
Sept., 1953

14. Ramey, R. A. — ON THE CONTROL OF M/A
AIEE Transactions, Vol. 70,
Part II, pp 2124
1951

15. Ramey R. A. — ON THE MECHANICS OF M/A OPERATION
AIEE Transactions, Vol. 70
Part II, pp 1214
1951

16. Ramey, R. A. — THE SINGLE CORE M/A AS A COMPUTER ELEMENT
Comm. and Elect., No. 4, pp 442
Jan., 1953

17. Rayleigh, Lord — NOTES ON ELECTRICITY AND MAGNETISM
III - ON THE BEHAVIOR OF IRON AND STEEL UNDER THE OPERATION OF FEEBLE MAGNETIC FORCES
Philosophical Magazine, Series 5
Vol. 23, pp 225
March, 1887

18. Scorgie, D. G. — FAST RESPONSE WITH M/A
AIEE Technical Paper 53-336
July, 1953

19. Van Allen, R. L. — A M/A FOR SYNCHROS
Comm. and Elect., No. 10, pp 749
Jan., 1954

APPENDIX

GAIN COMPUTATION

Voltage gain, $G = N\sqrt{1 - X^2}$

For the examples discussed the symbols have the following values. Meanings are also given although they are listed in the Table of Symbols and Abbreviations.

N	Turns ratio		1.5

$$X = \frac{H_c L R_{ct}}{K N_c^2 \left[1 - \frac{E_s}{E'_{ac}}\right]}$$

H_c	Coercive force per inch at 400 cycles		1.1
L	Mean length of flux path in inches		3.73
R_{ct}	Total control circuit resistance		510
K	Peak voltage per turn		.047
N_c	Control winding turns		800
E_s	Peak signal voltage	1st example	7.07
		2nd example	28.2
E'_{ac}	Peak control circuit supply voltage		39.5

Example 1.

$$X = \frac{1.1 \times 3.73 \times 510}{.047 \times 800 \times 800 \times \left[1 - \frac{7.07}{39.5}\right]} = .085$$

$$G = 1.5\sqrt{1 - .085^2} = 1.5\sqrt{1 - .0072} = 1.495$$

Example 2.

$$X = \frac{1.1 \times 3.73 \times 510}{.047 \times 800 \times 800 \times \left[1 - \frac{28.2}{39.5}\right]} = .245$$

$$G = 1.5\sqrt{1 - .245^2} = 1.5\sqrt{1 - .06} = 1.453$$

Thesis
W31
28477.
Weaver
Use of the single core magnetic amplifier in a magnetic timer circuit.